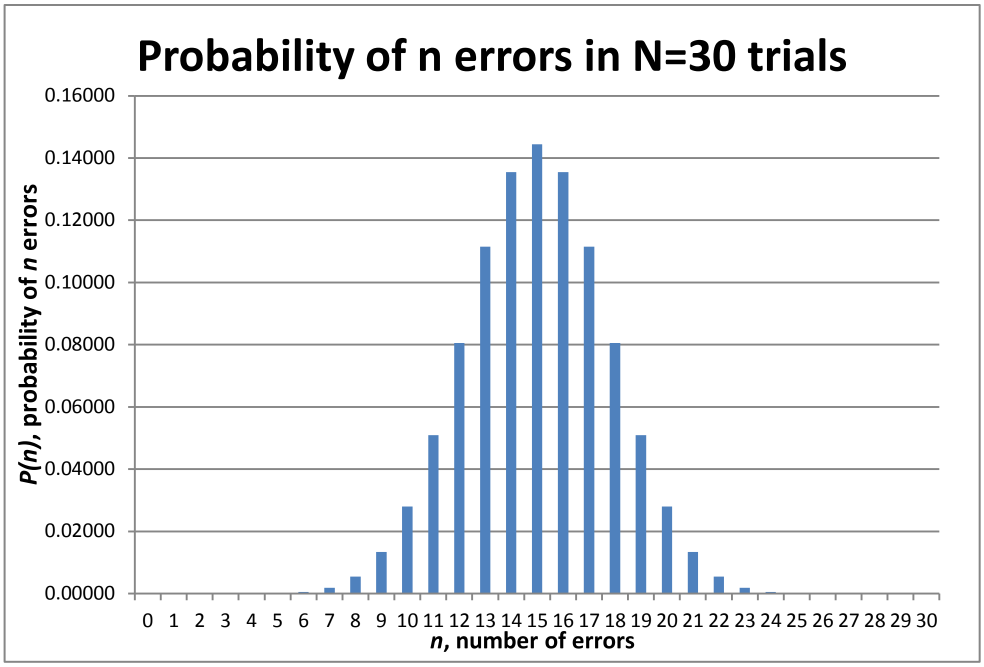
Consider the bar chart of probabilities of the binomial distribution with $N = 30$ and $p = ½$. The chart shows us all the possible values of $P(n)$, the probability of n errors in $N = 30$ trials. For example, if we compute $P(n=15)$ using the formula for the binomial distribution, we would find that $P(15) = 0.14446$. We also see that the height of the bar at $n=15$ is 0.14446. The x-axis depicts n, and we can see that it ranges from 0 to 30, and the y-axis is the probability $P(n)$, a number between 0 and 1. Recall that in our example of propagating errors, we were counting the number of errors (ε) after a certain number of steps or factors. Thus n is equivalent to the number of errors, and N is equivalent to the number of steps or factors. As we can see, the shape of this discrete distribution begins to resemble the familiar bell curve shape of the normal distribution.



Consider now a continuous model of measurement error. Suppose again that we are going to measure airspeed, x, with some transducer. Suppose further that at each step in the measurement process we can have fractional errors. In other words, we measure 202.5 when the truth is $x = 200$ knots. Here we have as the error term $\varepsilon = 2.5$, as an example. This is more like the physical reality than the binomial example above. To describe it adequately, we need the following additional definition.

Definition: Normal Distribution

The bell curve is formally known as the *normal distribution* and is the continuous probability distribution given by the probability density function below, where μ, the mean, and σ, standard deviation, are given.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Practically speaking, the parameters μ and σ, help us define the shape of the bell curve, whether it is tall and skinny or short and fat, for example.[1] There are two ways to observe our measurement error. We could plot a bell curve centered at 200 knots, the truth value, or we could subtract our measured value from the truth value and obtain an error term, $\varepsilon = 202.5 - 200 = 2.5$. In this second case, our curve would be centered at 0.

When we plot flight test data, we normally plot the raw values, so the former may occur more naturally. Additionally, plotting error terms is not always analytically tractable, and thus it is advantageous to plot the raw data. However, strictly speaking, it is the error term that we model with a normal distribution, not the airspeed term. Therefore, one must apply great care to avoid the mistakes in reason caused by a misunderstanding of what data are actually normally distributed.

This has been a very quick introduction to noise and the normal distribution, a vital tool that we use to model noise in flight test data, and we will close with one example that uses this tool: RNAV and RNP.

[1] See http://en.wikipedia.org/wiki/Normal_distribution for examples of normal distributions with different shapes.

Flight test is the place where validation of the accuracy of navigation systems occurs by comparison to a truth position source, and understanding the normal distribution is essential to applying the definitions of RNAV and RNP.

Figure 1 is from an FAA advisory circular and illustrates the definition of RNAV. Ninety-five percent is a common value used in many confidence intervals, based on approximately two standard deviations from the mean, and it is this concept used in the definition of RNAV. We won't take the time to discuss the subtleties of confidence intervals here, however.

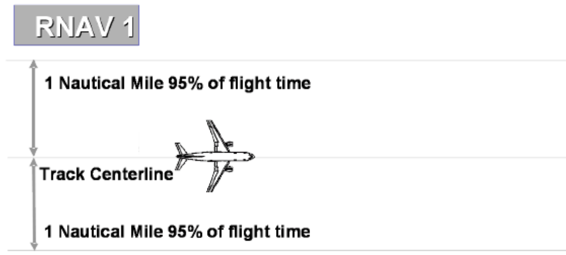


Figure 1 – RNAV lateral position error

In this second illustration, also from an FAA advisory circular, we see the definition of RNP. Again, confidence intervals are brought to bear on the navigation position, but this time, we see a much higher level of confidence, one exceeding three standard deviations from the mean.

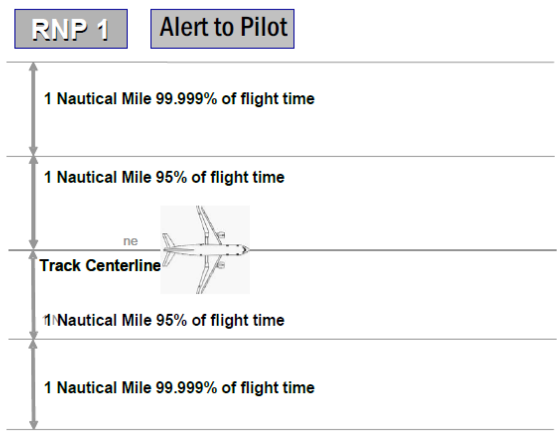


Figure 2 – RNP lateral position error