DEALING WITH THE WIND: AN ANALYSIS OF THE TURN REGRESSION AIRSPEED CALIBRATION TECHNIQUE

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ABSTRACT

A simple project to determine position corrections for the author's Bearhawk morphed into an investigation of the assumptions and sources of uncertainty in two airspeed comparison flight test techniques. This investigation led to the realization that the "constant wind" assumption in the Cloverleaf flight test technique really meant time-invariant and location-invariant winds, which in practice is too idealistic. Continued investigation into how uncertainty in the wind had resulted in less than desirable results from the Cloverleaf technique in the past led directly to a justification for the structure of the Turn Regression technique. The robustness of the Turn Regression technique in the face of time-invariant but location-varying winds is demonstrated. The effects of sample size on the results are discussed, as is the importance of flying a complete turn. In the end, actual position corrections were determined.

BACKGROUND

Since 2009, I have been flying my homebuilt Bearhawk. Because it is an Experimental Amateur Built aircraft, not a certificated aircraft, and because it has a unique Pitot-static installation, no altitude or airspeed corrections were available. Over five years of experience flying the aircraft, comparing indicated true airspeeds with GPS ground speeds seemed to imply that any errors were small. Even so, as the USAF TPS "Pitot-statics Guy" I felt the need to actually determine what those corrections were.

Having taught Pitot-statics (Air Data System Calibration) at the USAF TPS for 19 years and being involved with several Air Data System Calibration projects, I had seen first-hand the good points and the bad points of many different calibration techniques. Those that seemed to work well required a fair amount of infrastructure that would not be readily available to me to test my own airplane. The techniques that didn't require a lot of infrastructure didn't give satisfying results. There seemed to be a lot of uncertainty in the results, but I had no way to quantify that uncertainty.

However, there was a new technique that I had been peripherally involved with developing that seemed to offer promise. Upon trying it, the results were so much better than I had hoped for that it led to a full-fledged investigation into why it was so much better.

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TEST AIRCRAFT DESCRIPTION

Testing was accomplished on a Bearhawk (figure 1), designed by Robert Barrows. This example was a scratch-built Experimental Amateur Built aircraft, first flown in 2008. The four seat aircraft was powered by a Lycoming O-540, rated at 260 horsepower at sea level. Maximum takeoff gross weight was 2700 pounds.

Figure 1. Bearhawk

The Air Data sensor was an AN5816 Pitot-static tube mounted on a boom in front of the leading edge of the left wing. A Pitot-static tube was chosen to put the static ports in relatively undisturbed airflow, to avoid the expected errors of venting the instruments into the cabin, and to avoid the trial-and-error process of locating a suitable position on the fuselage. The Pitot tube opening was 40 percent chord in front of the leading edge. This positioning was relatively comparable to other aircraft with wing boom mounted Pitot (or Pitot-static) tubes, as shown in table 1.

Table 1. Historical Pitot Boom Lengths

Aircraft	Distance from Pitot Tube to Leading Edge
Cessna 195	28% chord
Bell P-39	52% chord
Curtiss P-40	55% chord
Republic P-47	30% chord
Focke Wulf FW-190	57% chord
Mitsubishi Zero	51% chord

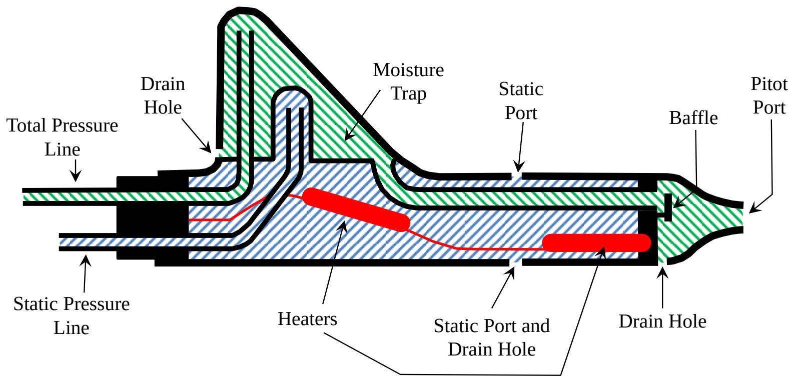


Figure 2. AN5816 Pitot-Static Tube Cutaway

The AN5816 Pitot-Static Tube had an interesting design, recognizable by the “shark fin,” shown in figure 2. The “shark fin” area was actually a moisture trap for both the total pressure and static pressure tubes, which with a baffle and drain holes was very effective at keeping water out of the instrumentation tubes.

INSTRUMENTATION

Data were recorded using the recording capabilities of the installed avionics. The Dynon Avionics EFIS D-10A (figure 3) recorded data for up to two hours at 1 Hertz. The data were downloaded to a laptop computer after the flight. The parameters used in this investigation are shown in table 2. Note that the GPS data recorded by the D-10A was supplied by a WAAS enabled Garmin GNS-480, which was interconnected with the D-10A.

Figure 3. Dynon Avionics EFIS D-10A

Table 2. Collected Parameters and Sources

Parameter	Source
Pressure Altitude	EFIS D-10A
Indicated Airspeed	EFIS D-10A
Heading	EFIS D-10A
Air Temperature	EFIS D-10A
GPS Ground Speed	GNS-480
GPS Ground Track	GNS-480
Other Parameters not used	EFIS D-10A

FLIGHT TEST TECHNIQUE SELECTION

Since I was funding this Pitot-static calibration myself, I was very interested in getting acceptable results at minimum cost. I was looking for a method that would have these desired properties:

1) **Minimal special instrumentation**. I didn't want to have to modify the airplane if I didn't have to.
2) **Minimal external equipment**. I didn't want to require large amounts of range infrastructure, such as a Tower Flyby range, which I couldn't get access to or didn't have the means to build.
3) **Easy to fly**. I wanted to be able to fly the points myself, rather than have to find a steely-eyed "Golden Arm" Test Pilot. Another acceptable option was to let the autopilot fly the maneuver.
4) **Quantifiable Uncertainty**. One of the big problems over the years has been that Pitot-static calibration techniques gave a result, but no information on how good that result was. Traditionally the only way around this was to collect large amounts of data and analyze the scatter.

Even though this would be a low budget program, there was not a requirement to limit ourselves to hand recorded data only. Because of the recording capabilities of the EFIS D-10A unit, methods that required a Data Acquisition System ("DAS required") were acceptable for consideration.

At the Air Force Test Center (AFTC), Pitot-static calibration tests have generally followed altitude comparison techniques. Table 3 summarizes the most common altitude comparison techniques with perceived advantages and disadvantages.

Table 3. Altitude Comparison Techniques

Technique	Advantages	Disadvantages
Tower Fly-By	Accurate Repeatable Easy to calibrate	One altitude only Subsonic only Needs range facility
Pace	Simple Multiple altitudes Can calibrate total pressure	Calibrated aircraft required Uncertainty passed along
Trailing Cone	Self contained Multiple altitudes	Takeup reel/Launch issues Lag May require calibration
Survey	Rapid Large airspeed range Supersonic Multiple altitudes	Weather balloon or Calibrated aircraft required

Looking at table 3, all of these techniques require some sort of special equipment not normally found on the test aircraft. In keeping with requirements 1) and 2) above, these techniques were rejected in hopes of finding something better.

The other approach to Pitot-static calibrations is airspeed comparison methods. Table 4 summarizes the most common airspeed comparison techniques with perceived advantages and disadvantages.

Table 4. Airspeed Comparison Techniques

Technique	Advantages	Disadvantages
Ground Speed Course	Simple Only external equipment is a known ground distance	Assumes constant wind Low altitude flight Requires tight airspeed control Slow process
Cloverleaf	No external equipment No heading measurement Tolerates slight variations in airspeed Multiple altitudes	Assumes constant wind Assumes constant temperature

The Ground Speed Course technique was very simple, but required very tight airspeed control. The uncertainty will go out the roof when the variation in airspeed is on the same order of magnitude as the error we are attempting to measure. Since the magnitude of both may be around 2 or 3 knots, this will be a problem. Thus, this technique violated requirement 3) above by not being "easy to fly."

The Cloverleaf technique looked promising, and has been used in the past by many programs. No requirement for external equipment was certainly a plus. Heading is tricky to measure, especially in the absence of a sophisticating navigation system, so the absence of a requirement to measure heading would **seem** to be a good thing. The ability to allow airspeed to vary slightly between legs would also be a plus. Sure, this method assumed a constant wind, but how tough can that be?

Even so, previous experience with using the Cloverleaf technique was not that satisfying. It is a deterministic method, meaning that it gives an answer, but no information about how good that answer is. That is to say, it returns no information about the amount of uncertainty in the answer. Flying multiple cloverleafs for the same airspeed tended to result in a large amount of data scatter, and the results didn't match that well with results from other techniques. All in all, my gut feeling was that the uncertainty in the results was significantly more than we wanted to believe it was. Thus, we took a deeper dive into analyzing the uncertainty inherent in the Cloverleaf technique.

CLOVERLEAF UNCERTAINTY ANALYSIS

Table 5 shows the possible sources of uncertainty and an estimation of their magnitude for the test aircraft. Note that only the bias (systemic) errors were considered. Precision (random) errors were assumed to be significantly smaller compared to the bias errors, and thus were ignored for this analysis.

Table 5. Cloverleaf Technique Uncertainty

Source of Uncertainty	Estimated Uncertainty
Total Pressure (airspeed)	±1 knot
Static Pressure (altitude)	±30 feet
Temperature	±1 degree Celsius
Heading (magnetometer)	±4 degrees
GPS Ground Speed	±0.19 knot
GPS Ground Track	< ±1 degree
Wind Speed	Large and Varying
Wind Direction	Large and Varying

The first six entries in table 5 seem to be reasonably small and manageable. The last two entries, namely the wind, seem a bit troubling.

The assumption requires a “constant wind.” Wind is a vector quantity, consisting of both speed and direction. If either one changes, it is no longer a “constant.” Another way to say “constant” would be to say “time invariant.” The best chance to have a truly constant wind for the Cloverleaf technique would be to take each of the three data readings at the same point in space. The flight path would look something like shown in figure 4.

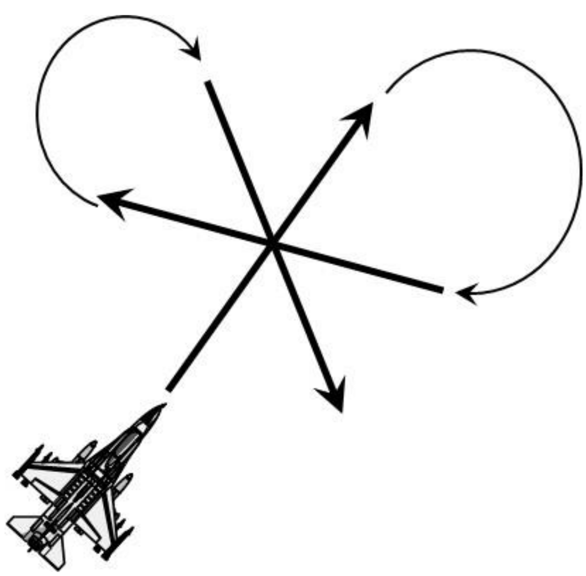


Figure 4. Cloverleaf flown through one point

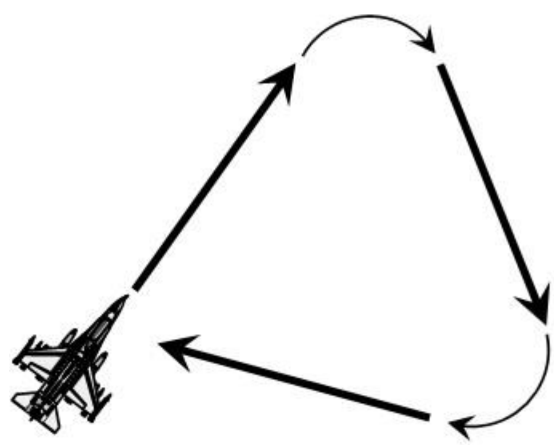


Figure 5. Cloverleaf flown as triangle

However, it is far more likely that the flight path will look like figure 5. Less time is spent turning (only 240 total degrees of turn instead of 480 total degrees), so it must be more “efficient.” Now the data collection happens at three distinct points in space. From a previous test, at F-16 speeds these locations can be as much as 20 nautical miles apart. Thus, our “constant wind” assumption has now changed from just “time invariant” to include “location invariant.” This is a far more restrictive condition, requiring the wind field to look something like figure 6. **NOTE**: All wind representations in this paper are shown as vectors, that is, showing the direction the wind is blowing “to”. This is in contrast to the traditional method of referring to where the wind is blowing “from”, and is done to show consistency with the true airspeed and ground speed vectors.

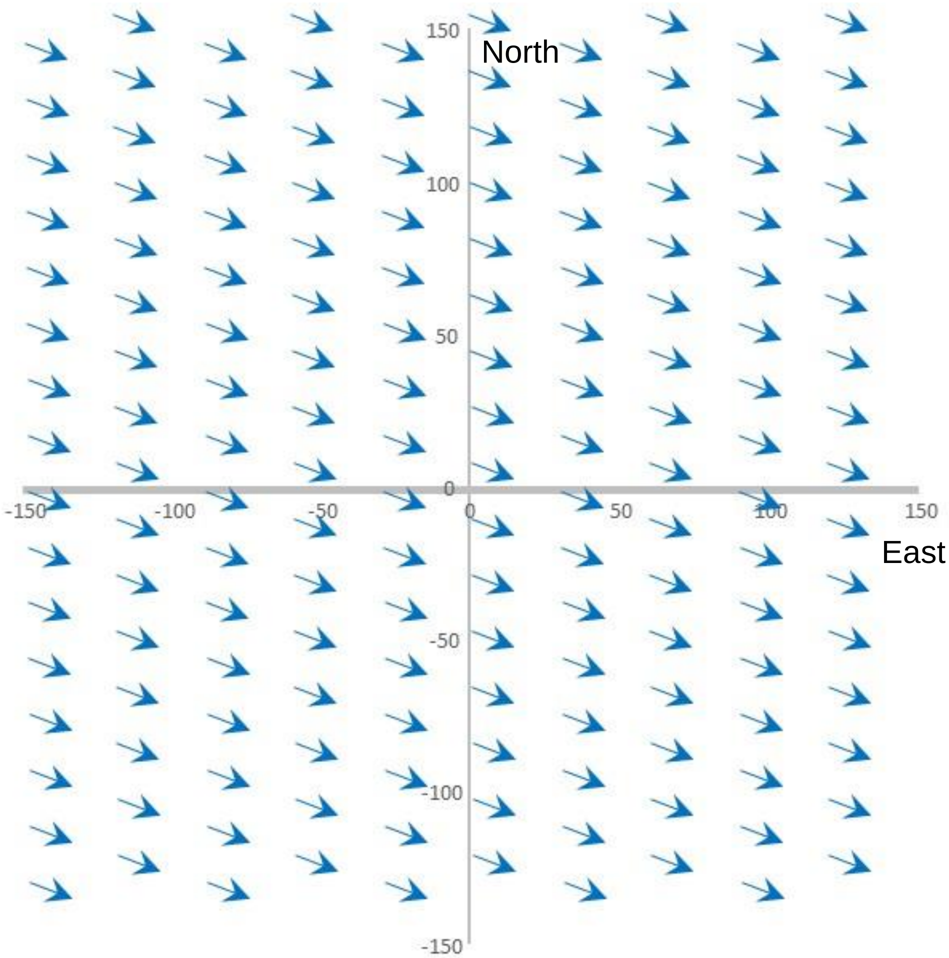


Figure 6. Time Invariant and Location Invariant “Constant” Wind

To get an insight to how the Cloverleaf data reduction works, let us for a moment assume a time invariant and location invariant wind field. Figure 7 shows fabricated “ideal” cloverleaf data where data for each leg were collected at identical true airspeeds and identical wind velocities. The ground speed vectors radiate out from the origin, with their base at the origin. The true airspeed vectors are placed to be head to head with the ground speed vectors.

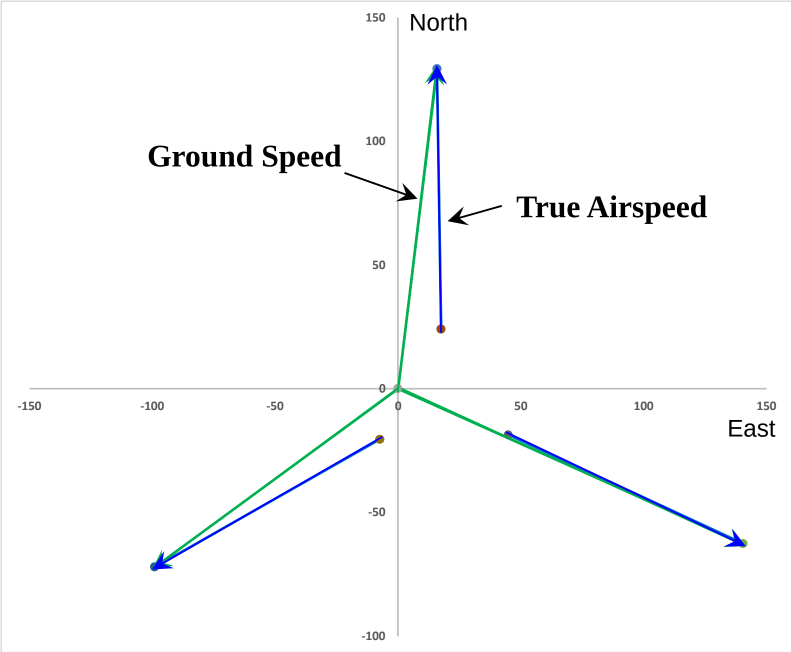


Figure 7. Ideal Cloverleaf data

Graphically speaking, the aim of the Cloverleaf data reduction is to extend each true airspeed vector magnitude (leaving the direction unchanged) by equal amounts (i.e. add a correction) until all three true airspeed vectors meet at one point, the tip of the wind vector. This is shown in figure 8.

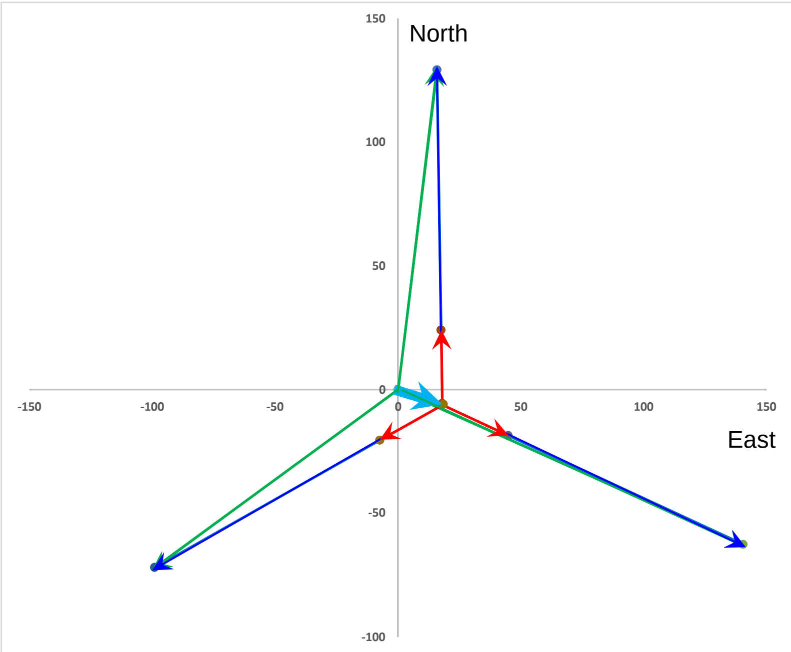


Figure 8. Ideal Cloverleaf data with true airspeed correction added, showing wind vector

As long as the assumptions hold, in this case an identical wind for each leg, the data reduction method and the Cloverleaf technique work. But how likely is it that the wind will be identical for each leg? Sadly, the answer appears to be "not very."

CHARACTERIZING THE WIND

Ask any person about a "constant wind" and they will probably tell you that they have experienced such a thing. All of us have stood in one location and felt a wind that did not change in speed or direction. This certainly implies a "time invariant" property at a single location. However, from our own experience of standing outside, it is very difficult to determine if the wind changes from one location to another because we can only be in one place at one time and we can't move very fast from one location to another.

There is a group that can talk about wind variations with location. Sailplane pilots (glider pilots) not only know that the wind can change with small changes in location, but also that air moves vertically as well as horizontally. In fact, they exploit these properties of the atmosphere to keep their gliders aloft for hours on end. Learning micro-meteorology is a necessary pursuit to become a successful sailplane pilot.

A simple analogy should make this clear. Consider the water flowing in a shallow, rocky stream. You can select many locations in the stream where the water flow vector at that location does not change with time. However, the speed and direction of the flow at any one point is different than the speed and direction at every other point. The flow is time invariant but not location invariant.

Calm winds at the surface are also misleading, as the Earth has a boundary layer. The wind can be calm at the ground, yet have significant speed a few hundred feet up. This can be seen as turbulence experienced right after takeoff or just before landing.

Temperature inversions can also be deceptive with respect to the wind. On a clear night, the ground radiates heat into space in the infrared wavelengths. This causes the ground to cool, which cools the air near the ground. Air higher up, transparent to the infrared radiation, does not cool. With cooler air near the ground and warmer air above, the temperature gradient is stable, so the air likes to stay where it is. The stable air will tend not to move, so the winds at the surface will be calm. Above the inversion layer, winds will continue to blow, but won't mix with the stable layer. This condition will persist until the sun warms up the ground, which in turn warms up the air and the vertical movement removes the inversion layer. After the inversion layer is removed, the wind will work its way down to the surface.

Even air well above the influence of terrain can be affected. Take a look at a Winds Aloft forecast. Any sudden changes in the wind direction or speed are indicative of wind shear. Wind shear begets turbulence, and turbulence is indicative of non-uniform winds. How many times have you been on a commercial flight and the pilot was apologizing for the turbulence?

Many other examples exist to show that winds can be non-uniform. In fact, the existence of weather is caused by uneven heating of the earth, and therefore the atmosphere.

FLYING CLOVERLEAF DATA IN NON-UNIFORM WIND FIELDS

Having established that a uniform wind field, such as shown in figure 6, is unlikely, what can we do to mitigate the uncertainty that non-uniformity brings?

To evaluate data reduction in real-world wind patterns, wind data were derived from data recorded on two separate test days. Data were collected while flying constant load factor (bank angle) turns at 5000 feet pressure altitude about eight miles northwest of Fox Field (KWJF) in the Antelope Valley. These wind data are shown in figures 9 and 10. For this presentation, the air track was circularized for clarity, such that the wind field would be apparent without being convoluted with the drift of the aircraft with the air mass. This is analogous to looking at true airspeed instead of ground speed. The base of each wind vector was located at a pseudoposition on a 100 knot radius circle, with the radial location determined by the heading plus 90 degrees. Graphically this pseudoposition approximates the actual location of the aircraft. The radius of the circle (100 knots) was arbitrarily chosen as a good match with the actual wind speeds. The length and direction of the wind vector corresponds to the actual wind calculated for that test point.

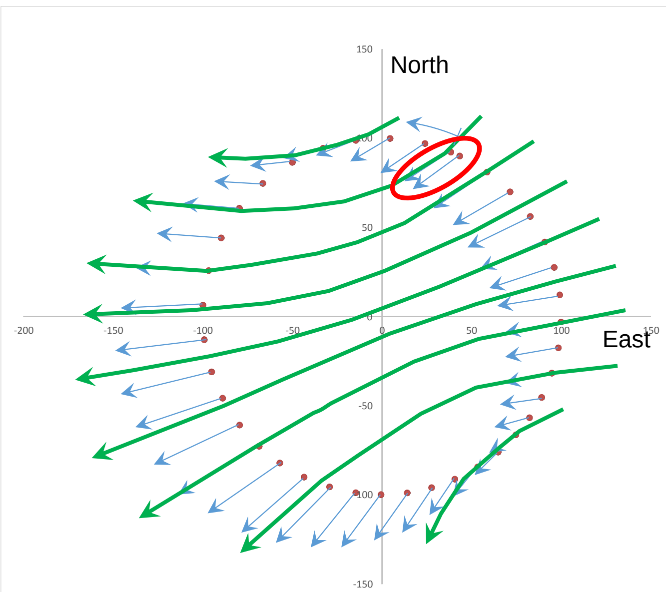


Figure 9. East Wind Pattern

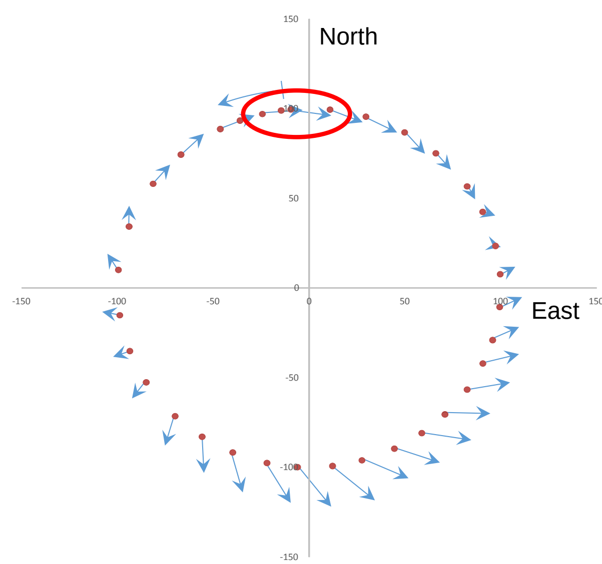


Figure 10. West Wind Pattern

Figure 9 shows a somewhat reasonable wind pattern with an average east wind. This turn was a left turn, and the ellipse identifies the beginning and ending points. Because the wind vectors at the first and last points are very similar, we can infer a time-invariance for the flow field. Much like fairing a curve through data points, an assumed wind field was drawn through the circle of data. This consistent wind field was believable as time-invariant, but the wind speed and direction certainly changed from location to location.

Figure 10 shows a wind pattern recorded at the same location, but with an average west wind. Once again, the turn was to the left, and the ellipse identifies the beginning and ending points. Because the wind vectors at the first and last points are very similar, we can infer a time-invariance for the flow field. However, it is not possible to draw a steady flow field from west to east that will match the vectors shown. It appears that there is a source in the middle of the circle flowing outward, but this is not possible. This strange wind pattern could possibly be explained by horizontal vorticity caused by the local terrain.

Why the difference? In the test area, the valley floor elevation was about 2300 feet MSL. With the test altitude at 5000 feet pressure altitude, or approximately 2700 feet AGL, the terrain was very flat to the east, so an east wind had very few perturbations from the terrain. However, on the west side were ridges in a "V" shape around the test area, with the tops of the ridges at or above the test altitude. It was quite reasonable for a west wind blowing over these ridges to set up wind shears and vorticity.

To investigate what happens to the Cloverleaf data reduction method when the constant (uniform) wind assumption breaks down, let us select three data points from figure 10. The selected data points are shown in figure 11, showing the true airspeed vectors and wind vectors. Note that the wind vectors for the three legs are significantly different.

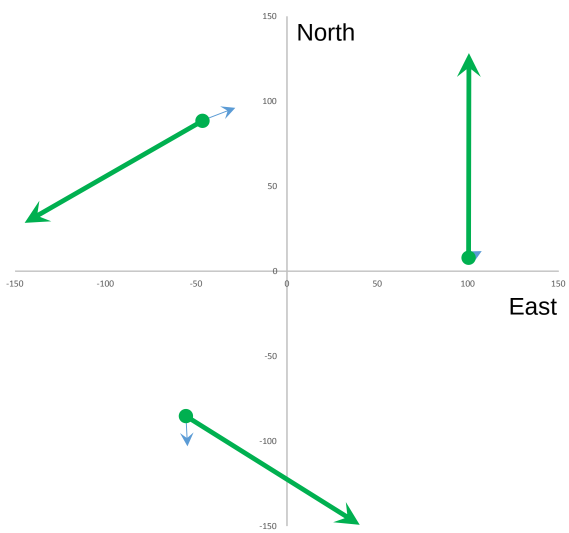


Figure 11. Simulated Cloverleaf legs

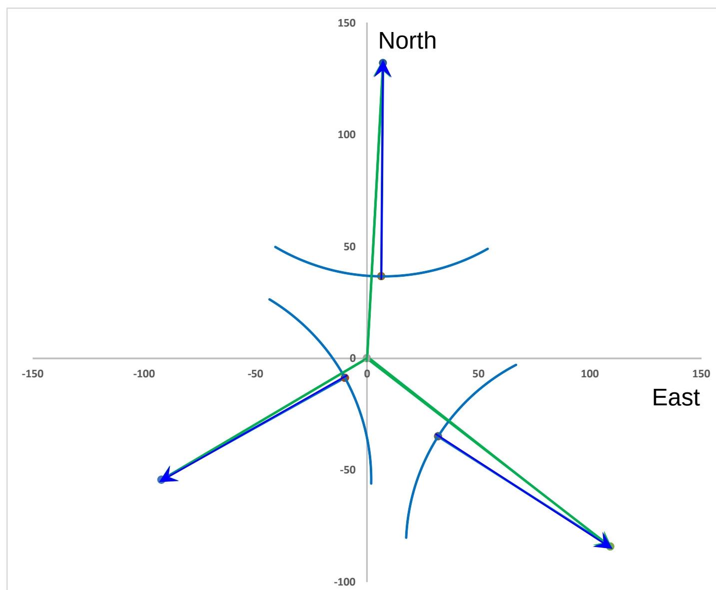
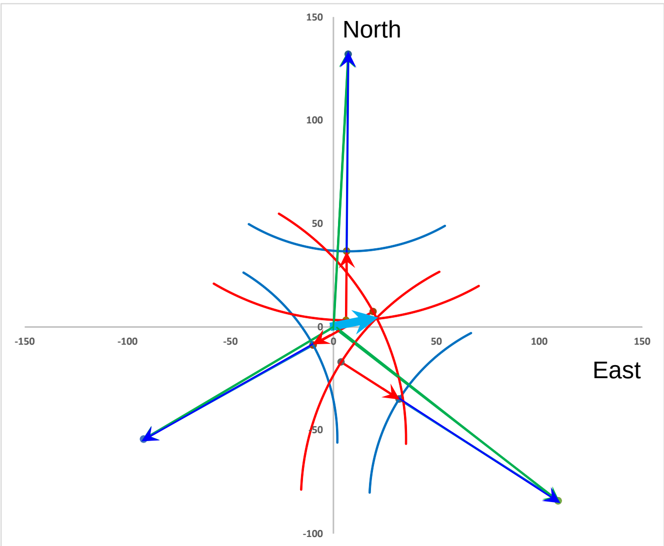


Figure 12. Cloverleaf data

Figure 12 shows the same data from figure 11 in the format of figure 7, with the magnitude of the correction exaggerated for clarity. The ground speed vectors radiate out from the origin, with their base at the origin. The true airspeed vectors are placed to be head to head with the ground speed vectors.

Earlier we said "Graphically speaking, the aim of the Cloverleaf data reduction is to extend each true airspeed vector magnitude (leaving the direction unchanged) by equal amounts (i.e. add a correction) until all three true airspeed vectors meet at one point, the tip of the wind vector." At least, that was the intent of the data reduction method. However, further analysis of what is going on in the math reveals that the analysis doesn't even consider the direction of the true airspeed vector. This was the perceived benefit of not having to measure heading angle. In figure 12, the magnitudes of the true airspeed vectors are represented by the arcs, centered on the heads of the ground speed vectors. A constant correction is added to the magnitude of each true airspeed vector until all three arcs intersect at one point. If the initial true airspeed vectors are of equal magnitude, the resulting point would be equidistant from the three heads of the ground speed vectors. The result is shown in figure 13.



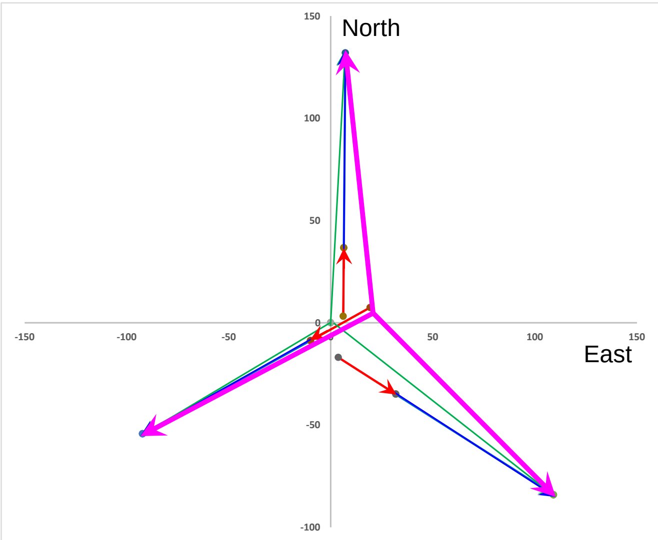
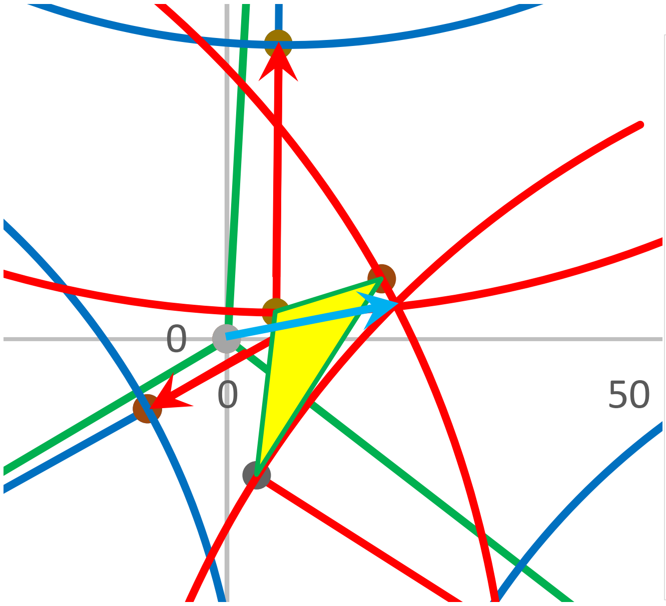


Figure 13. Cloverleaf solution (?)

Figure 14. Heading difference

Figure 14 shows the true airspeed vectors with corrections in their original direction from figure 13 plus the supposed solution for the true airspeed vectors from figure 13. It is troubling that the headings for the calculated solution aren't even close to the actual headings that were measured. Hmmm. When the uniform wind assumption is violated (actual wind vectors are not the same for each leg), the calculated headings can be significantly different from the actual headings. This could explain the less than satisfying results seen in using the Cloverleaf method in the past.

Figure 15 is an enlargement of the center of figure 13. This shows another curious problem with the Cloverleaf data reduction method in non-uniform wind. The corrected true airspeed vectors do not meet at one point. A triangle is drawn between the bases of these vectors. If the bases are supposed to meet at one point, and that point would be the head of the wind vector, shouldn't the head of the calculated wind vector at least fall inside of that triangle? In this case, as shown in figure 15, it does not.



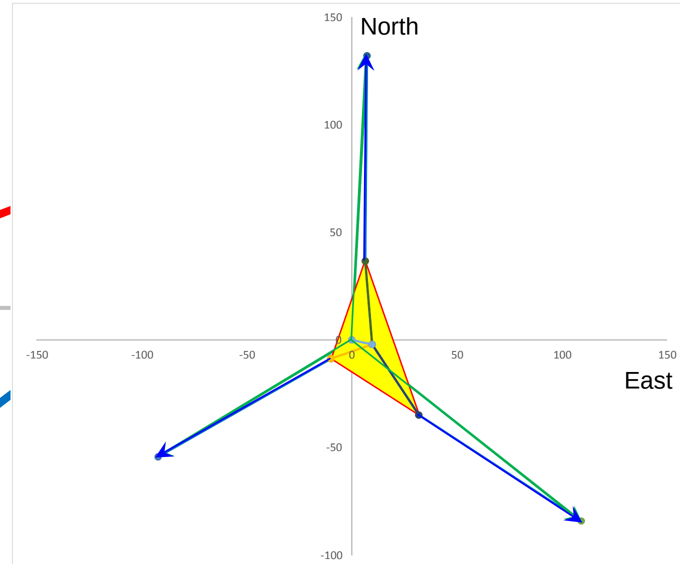


Figure 15. Figure 13 enlarged **Figure 16. Centroid of the triangle**

THERE MUST BE A BETTER WAY

The Cloverleaf data reduction method does lead to an exact mathematical solution for the data supplied. However, the major assumption that the wind vector was exactly the same for each leg of data is critical. If that assumption is violated, then the result starts to deviate from what would be considered "reality."

Further investigation of figure 15 suggests a different approach. Intuition suggests that if the three true airspeed vectors do not come to a single point to define the wind vector, then the wind vector tip should at least be somewhere in the triangle formed by the bases of the true airspeed vector. Since, when in doubt, we tend to "average" data, the equivalent in this case would be to find the centroid of the triangle and use this as the tip of the wind vector. The centroid is the location where the sum of the three distances from the centroid to the vertices is minimized.

As a constant correction is added to the magnitude of the true airspeed vectors, the resulting triangle will change, and the sum of the distances from the centroid to the vertices will change. When this sum of distances resulting from adding a correction is minimized, the correction solution is found.

Uncertainty theory tells us that we can reduce the precision error in a result by increasing the number of samples. Fortunately, the idea above is not limited to just three legs. Figure 17 shows what we would get for selecting 12 legs of test points from the wind field shown in figure 10. The wind vector tip would be at the centroid of the twelve bases of the true airspeed vectors, or the location at the minimum total distance from each of the bases. To find the value of the true airspeed correction, add a correction until the total distance from the centroid to each of the bases is minimized.

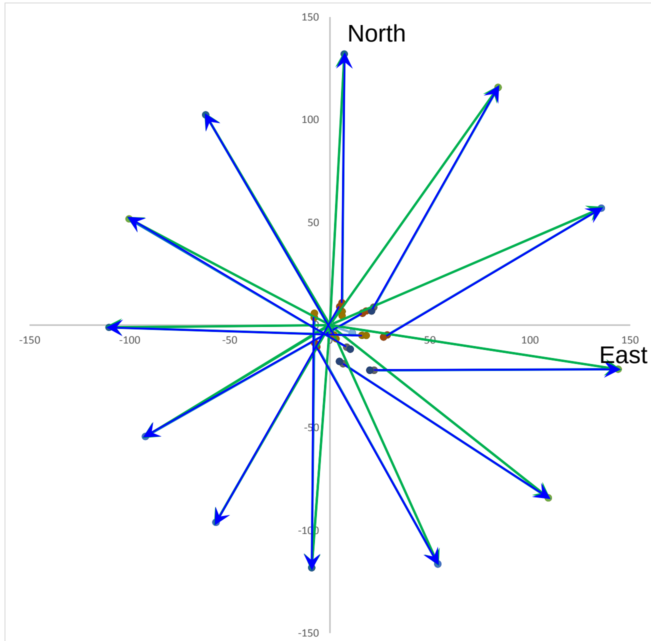


Figure 17. Twelve leg solution

By now you are probably thinking that this methodology sounds strangely familiar, and it should. The “distances” from the centroid to the bases of the true airspeed vectors are what mathematicians call “residuals.” A residual is the difference between a model prediction and the actual result. Said another way, it is the part of the observed result that can’t be explained by the system model. As for finding a solution that minimizes the sum of the distances, a Least Squares Regression analysis is all about minimizing residuals. Thus, our intuitive approach to try to improve the Cloverleaf data reduction results has led us right to the widely accepted approach of Least Squares Regression.

So how do we get lots and lots of samples? Simply fly a level turn, and as often as possible record true airspeed (usually from indicated airspeed, pressure altitude, and ambient air temperature), heading, GPS ground speed, and GPS ground track. In this analysis, this approach at 1 Hertz yielded 60 to 200 samples, depending on the turn rate.

But haven’t we seen this before? Why, yes we have! In 2010 Al Lawless suggested the Orbis method (reference 1), which determined the average wind by matching flight path circles. In 2011 Tim Jorris suggested “Statistical Pitot-Static Calibration Technique using Turns and Self Survey Method” (reference 2) which detailed the method to be discussed in the remainder of this paper. In 2015, Juan Jurado presented excellent results from a modified version of this technique (reference 3).

In summary, the system is modeled from the wind triangle for each data point, as shown in figure 18. Each vector is split into two components, such that each data point results in two equations. Figure 18 shows what portions of the equations can be filled with measured values and what portions are the unknowns.

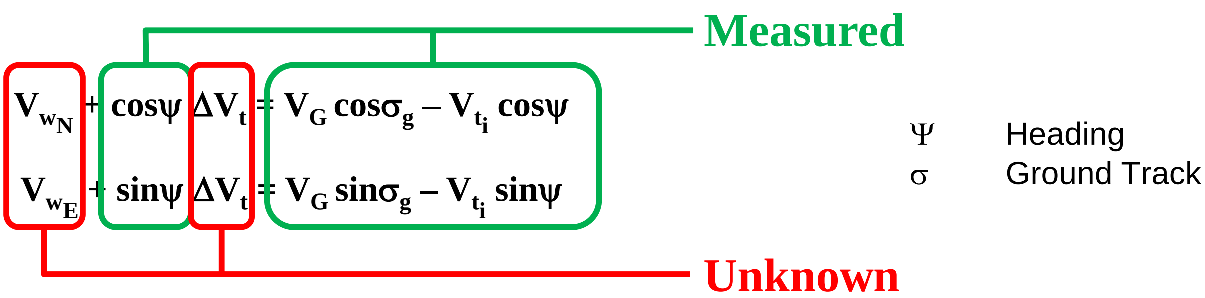


Figure 18. Wind Triangle Model Equations for a single data point

The equations in figure 18 can be recast into matrix form as

$$\begin{bmatrix} 1 & 0 & \cos\psi \\ 0 & 1 & \sin\psi \end{bmatrix} \begin{bmatrix} V_{w_N} \\ V_{w_E} \\ \Delta V_t \end{bmatrix} = \begin{bmatrix} V_G \cos\sigma_g - V_{t_i} \cos\psi \\ V_G \sin\sigma_g - V_{t_i} \sin\psi \end{bmatrix}$$

[A] [B] = [C]

In this form, the [B] matrix contains the unknowns. The measured values fill out the [A] and [C] matrices. For each data point added, the [A] and [C] matrices will add two rows. The [B] matrix will be unchanged. Using the Regression Add-In in Microsoft Excel, the [A] matrix will be the “Input X Range” and the [C] matrix will be the “Input Y Range”. Note that the model contains no constant term, so “Constant is Zero” will need to be checked.

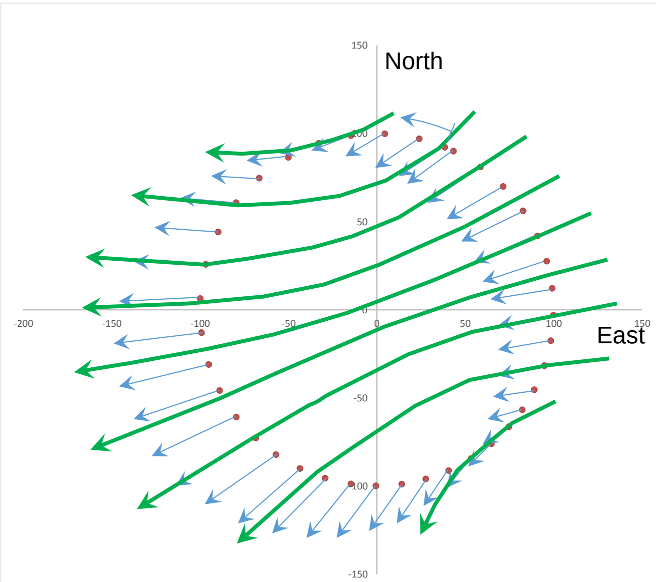
UNCERTAINTY ANALYSIS OF THE TURN REGRESSION METHOD

So far we have shown that the Cloverleaf data analysis method does not handle uncertainty very well. In an effort to handle the uncertainty better, we led ourselves to using a Least Squares Regression method. Using actual flight test data, let us try to gain an understanding of how the Turn Regression method can tolerate uncertainty and the estimated accuracy of the results.

The biggest source of uncertainty, as identified in table 5, was the wind. How much uncertainty is introduced by the wind will depend on how well we can characterize the wind. The challenge here is that there is no effective way to collect truth data on the wind over a large area. The next best thing we can do is to see if our measurements trend with our assumptions. In this case, we will assume that the wind is steady, or time-invariant, in the short term, possibly changing slowly in the long term. The wind is allowed to change with location, but will remain the same at any particular location.

The approach was to fly eight airspeed calibration turns in the same general location, with airspeeds varying from 74 KIAS to 114 KIAS. Each turn resulted in an average wind direction and speed output. The hypothesis was that if this method is tolerant of the uncertainty introduced by winds changing with location (but not with time), then the calculated average wind direction and speed should not change with time or with changing airspeed.

Figure 19 shows the wind field measured during the turn at 114 KIAS, and is in fact the same wind field as shown in figure 9, repeated here for easy comparison with figure 20. Figure 20 is the wind field measured at 100 KIAS, flown 9 minutes after figure 19. Over this short time span and measured at a different airspeed, the wind field looks pretty much unchanged, as would be expected.



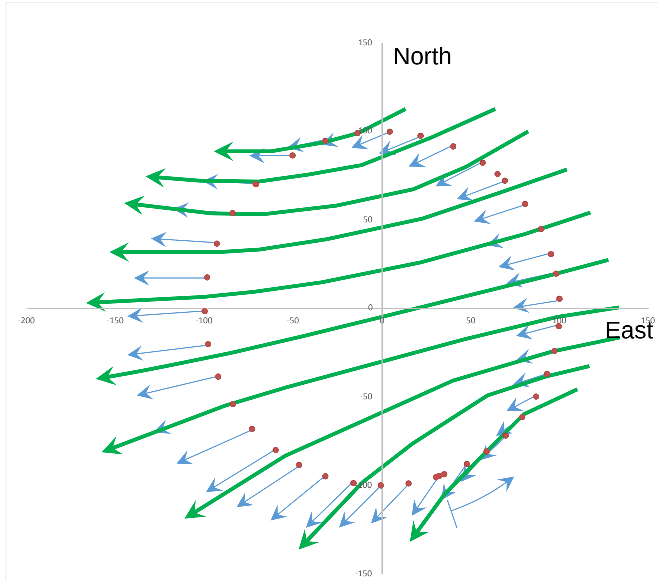


Figure 19. Wind Field at 114 KIAS

Figure 20. Wind Field at 100 KIAS

Figure 21 shows the measured average wind direction and wind speed calculated by the turn regression method by time of day and changing airspeed. If the wind variations introduced large uncertainty into this method, the results should be randomly scattered with no apparent pattern. As it is, the wind direction and wind speed seem to trend in a way that could be expected over a span of 23 minutes. The variation of airspeed for each sample does not have any obvious effect on the wind measurement. These results boost our confidence that the wind is being properly and consistently characterized.

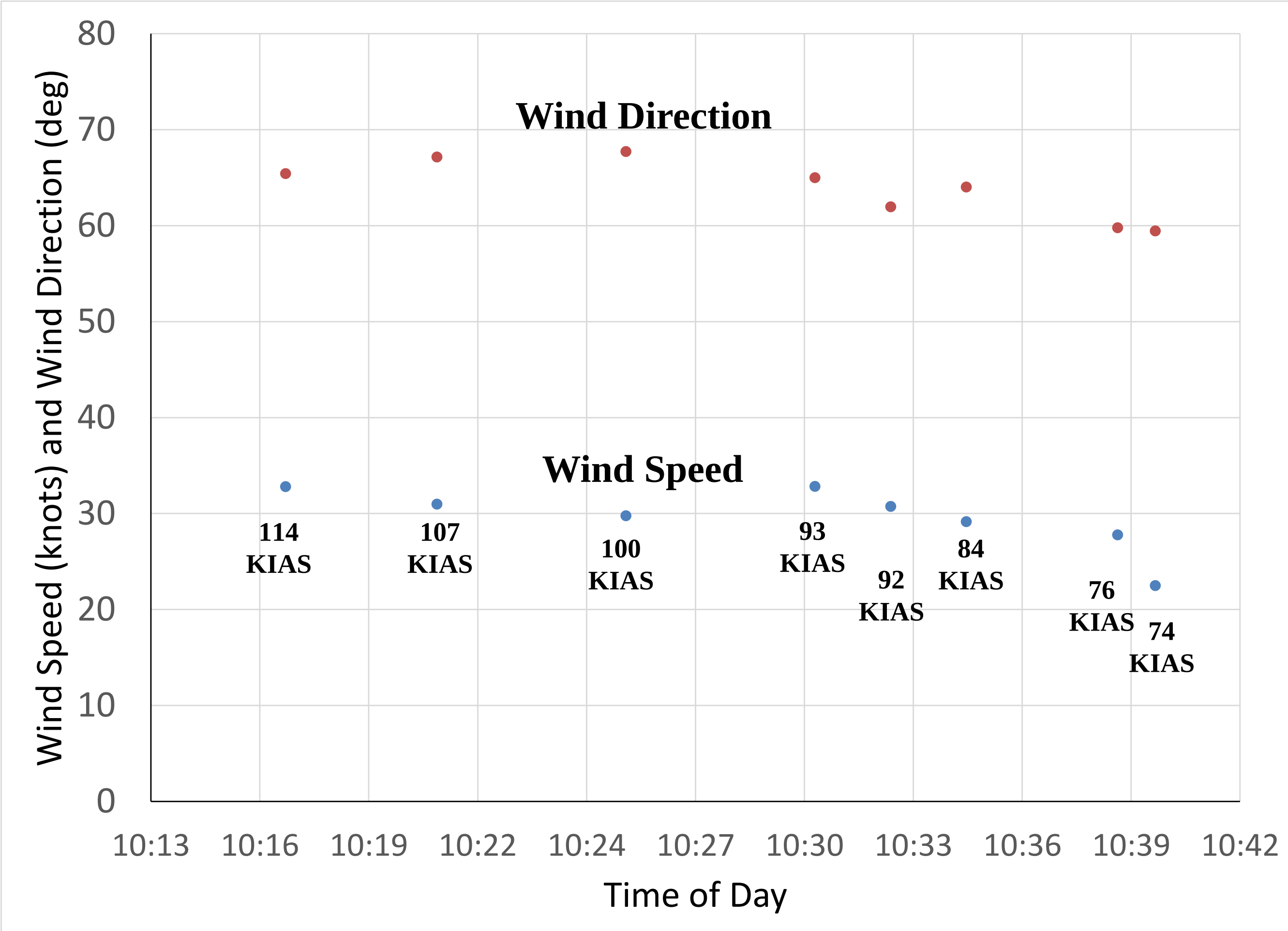


Figure 21. Time history of Wind Direction and Speed

The wind pattern for these data, as shown in figures 9 and 19, appears to be relatively well behaved. What would happen if the wind pattern was much more poorly defined, as in figure 10? To investigate this, consider the calculated true airspeed corrections as shown in figure 22. The calculated true airspeed corrections are shown by indicated airspeed as the diamond symbols. The error bars indicate the 95 percent confidence interval on the true airspeed correction as calculated by the turn regression data. The data points not marked by ellipses correspond to the data points shown in figure 21, collected in wind conditions similar to that shown in figure 9. The data points marked by ellipses were collected in the squirrelly wind conditions shown in figure 10. Even with the much more poorly behaved winds of figure 10, both of these data points fall right in with the other data, and in fact their confidence intervals are of similar magnitude. This similarity would indicate that the large number of samples (around 200) allows the turn regression method to manage the uncertainty well and still give good results.

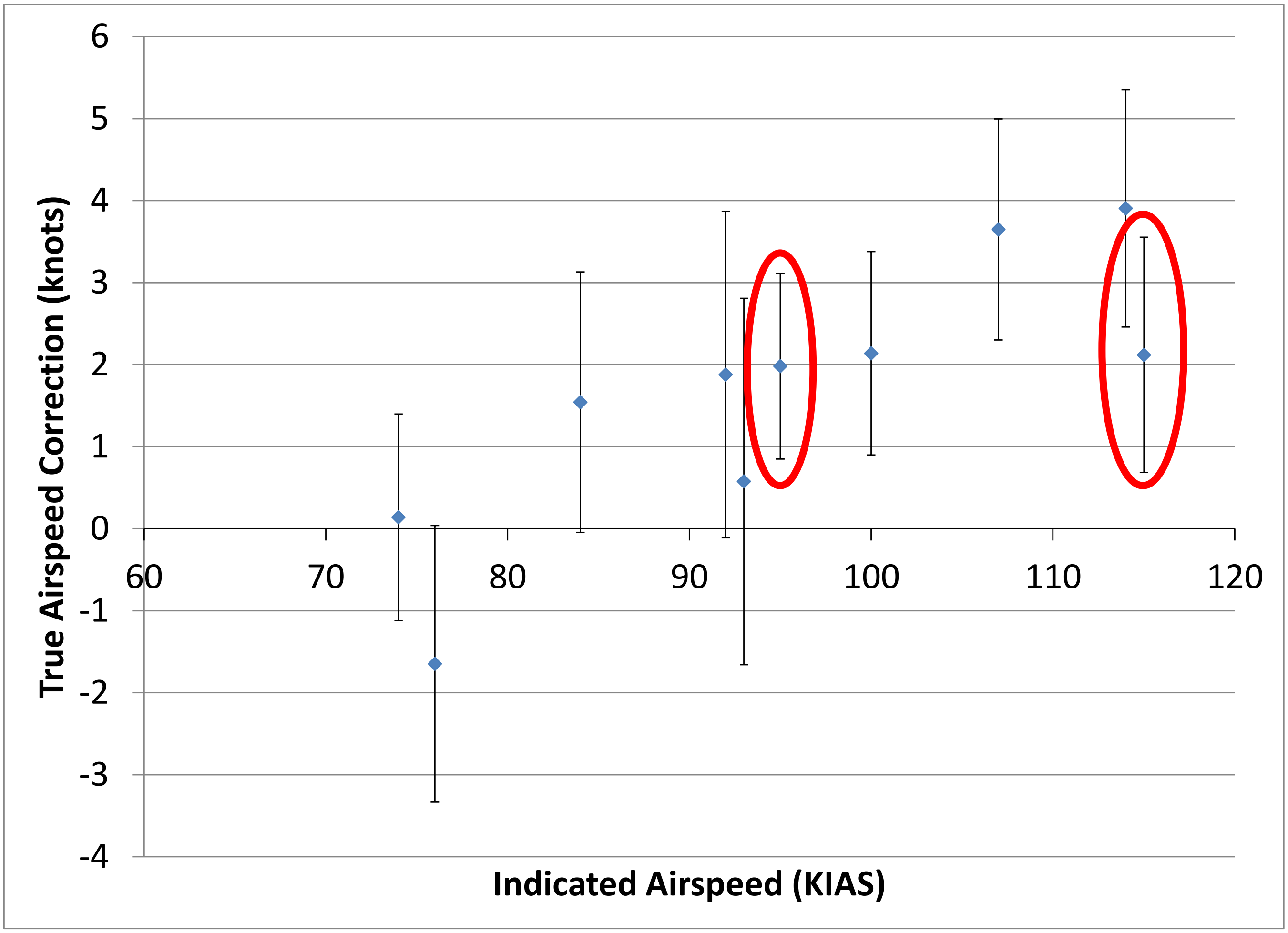


Figure 22. True Airspeed Correction Results

As introduced in figure 22, one of the major strengths of the Turn Regression method is that it supplies statistics on uncertainty, specifically the size of the confidence interval. Traditional methods such as the Ground Speed Course and Cloverleaf are deterministic, giving a result with no indication as to the uncertainty involved. The Flight Test industry has lived this way for years, so just how bad could it be? We can use the Turn Regression data reduction methods to investigate this question.

Figure 23 takes the data from figure 17 and breaks it up into four sets of three legs. Each set of data consists of three legs approximately 120 degrees apart in heading to give the best possible results. These data were run through the Turn Regression data reduction with the results shown in table 6.

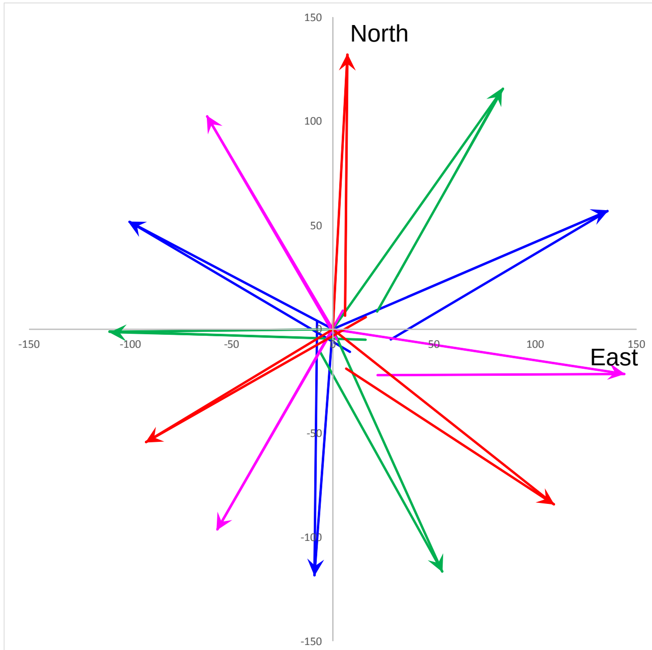


Figure 23. Four simulated cloverleafs

Table 6. Cloverleaf Uncertainty

Sample (n=3)	ΔV_t	p value	Confidence Interval
A	1.8	0.657	[-21.3 24.9]
B	2.8	0.677	[-23.7 29.2]
C	2.1	0.738	[-27.1 31.3]
D	2.3	0.682	[-25.8 30.3]

The first thing to notice in table 6 is that the values for the true airspeed correction (ΔV_t) don't vary much. Since each of these sets of data are taken at the same airspeed and flight condition, the resulting true airspeed correction should be the same. The spread of values in table 6 is less than a knot, which is considered too small to be significant.

The p value, also known as "Significance F", is the probability that the model does NOT explain the variation in the output. That is, the likelihood that any fit between the model and the data is purely by chance. Ideally, the p value should be small. A generally accepted p value is less than 0.05, which would imply a 95 percent probability that the model does explain the variation in the output. Looking at the p values in table 6, the numbers are significantly greater than 0.05, which means that the model is statistically questionable, even though the result seems reasonable. A lack of statistical significance is somewhat expected since there are only three data points in any analysis.

The confidence interval is a range that the statistics predict a 95 percent probability that the mean value of the output will fall in. This is different than the prediction interval, which is a range with a 95 percent probability that the next output value will fall in. A smaller confidence interval is better, but the intervals shown in table 6 (±23 knots) are ridiculously large compared to the desired uncertainty.

The statistics look really bad, and call into question how we've survived this long not knowing about this. Let's try adding more data samples to reduce the precision error.

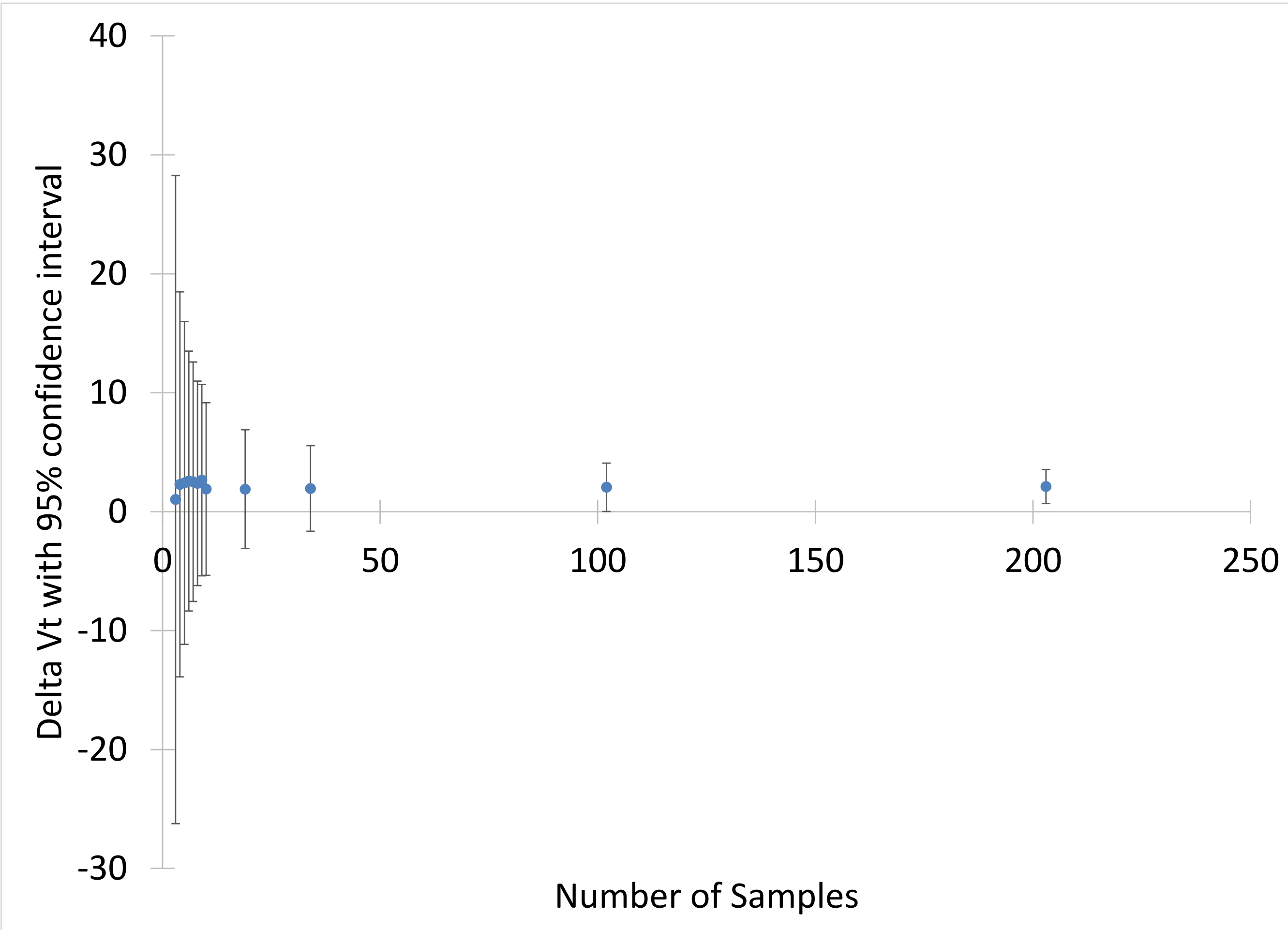


Figure 24. Confidence interval shrinking as number of samples increases

Figure 24 was created from one set of data by running the regression on different sample sizes from one data set. The samples were always selected to be evenly distributed around the complete circle. We see in figure 24 that indeed the confidence interval on the true airspeed correction shrinks rapidly as the sample size is increased, as small as ±1.5 knots at 200 samples. Also notice that the calculated true airspeed correction magnitude stays essentially constant as the sample size is increased. It would seem that as long as the sample headings are evenly distributed around the circle, the system of equations is well behaved, giving answers very close to the statistically significant answer. This would be the answer to why we have been able to get acceptable answers with much smaller sample sizes, such as the three legs of the cloverleaf.

The real benefit of the turn regression method and its statistically large sample sizes is that the resulting confidence intervals give us a means to quantify the overall uncertainty in the result, aggregating the effects of all of the uncertainties in table 5. The smaller the confidence interval, the smaller the uncertainty. However, it is possible to get too much of a good thing. If the samples are too close together, we run into serial correlation issues, where each sample isn't really independent of the sample before or after it. In this study, the samples were collected at 1 Hz, which seemed to work out okay. If, for instance, the samples were collected at 60 Hz, the data would probably need to be decimated to preserve independence.

So having more samples is generally good, and data acquisition systems (DAS) can typically record more than enough samples in a single turn. This doesn't even require the high dollar orange box DAS. WAAS GPS units are widely available, and recording Electronic Flight Information Systems (EFIS) are available which can record sufficient data. But what if we don't have a recording system? How many samples would we need?

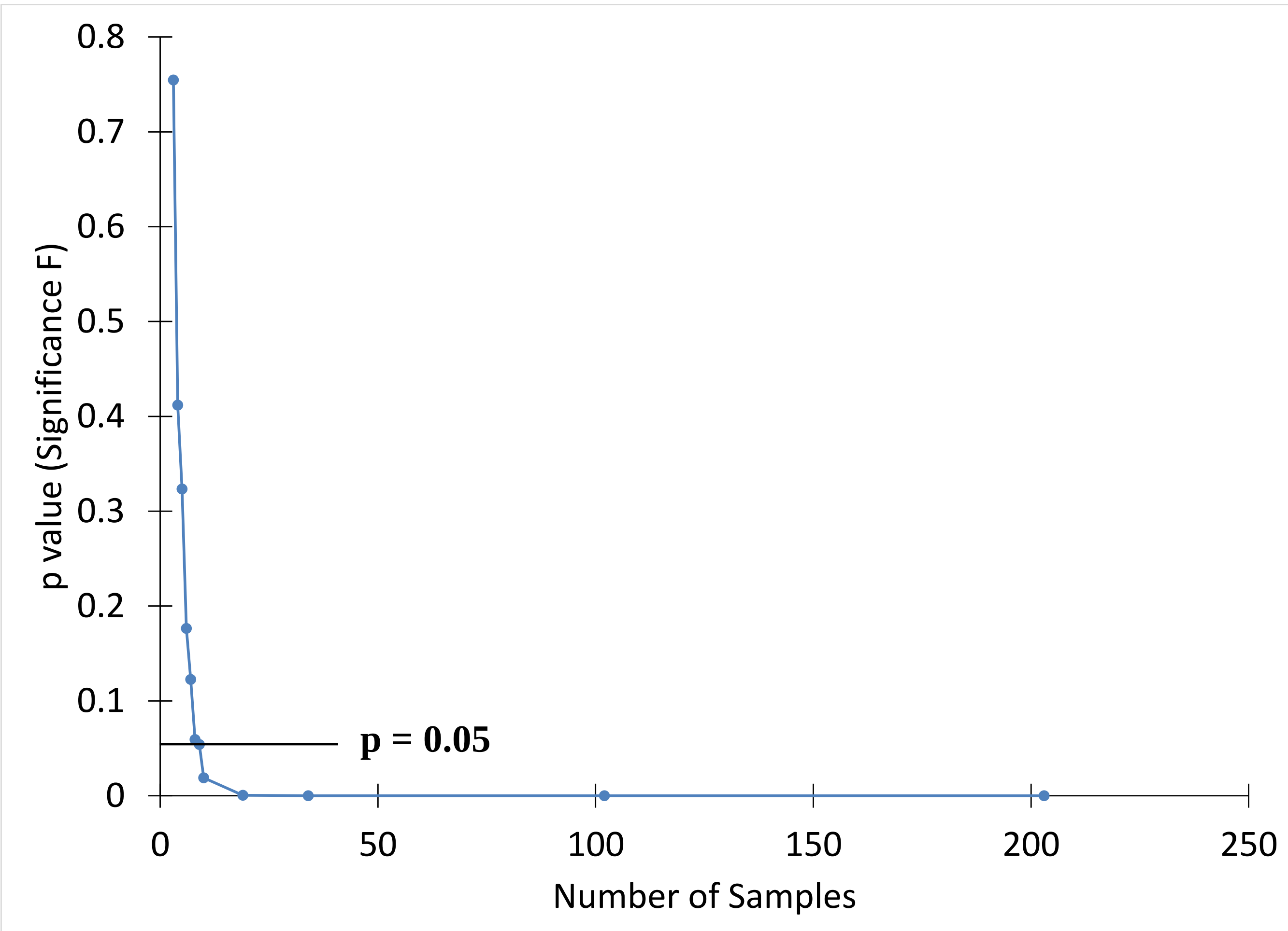


Figure 25. p value change as number of samples increases

Figure 25 was created with the same data as figure 24. This figure shows that to have a p value less than 0.05, at least 9 to 10 samples will be required. Figure 24 shows that with 10 samples the confidence interval is ±7.3 knots, which is still rather large. As noted earlier, the results are well behaved, and the value for the true airspeed correction does not change much as the sample size changes.

Up until now, all data points (samples) were evenly distributed around the turn circle. Would it save time to just fly part of the circle? It turns out this would be a very bad idea. Since all of the headings are more or less in the same direction, the solution wanders as the statistical independence of the measurements comes into question. Consider table 7.

Table 7. Partial Turn Results

Degrees of Turn	ΔV_t	p value	Confidence Interval
360	2.1	1.36E-44	[0.7 3.6]
180	-3.2	9.87E-15	[-6.0 -0.4]
90	-16.9	2.62E-25	[-19.6 -14.1]
45	-21.2	2.62E-43	[-23.4 -19.1]

The p values and confidence intervals look really good, but look at the true airspeed correction. It changes wildly even though the same data are used for each case. The 45 degree turn data are just the first 1/8th of the 360 degree turn data. What's going on?

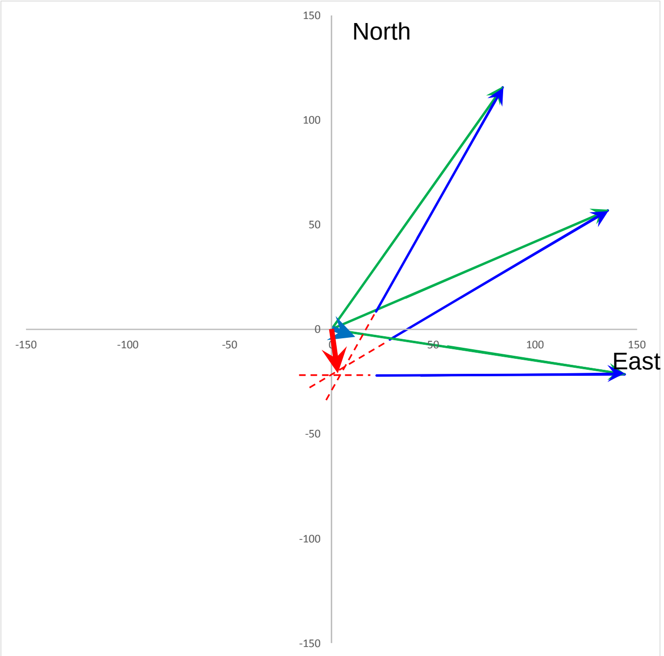


Figure 26. Ill-posed solution

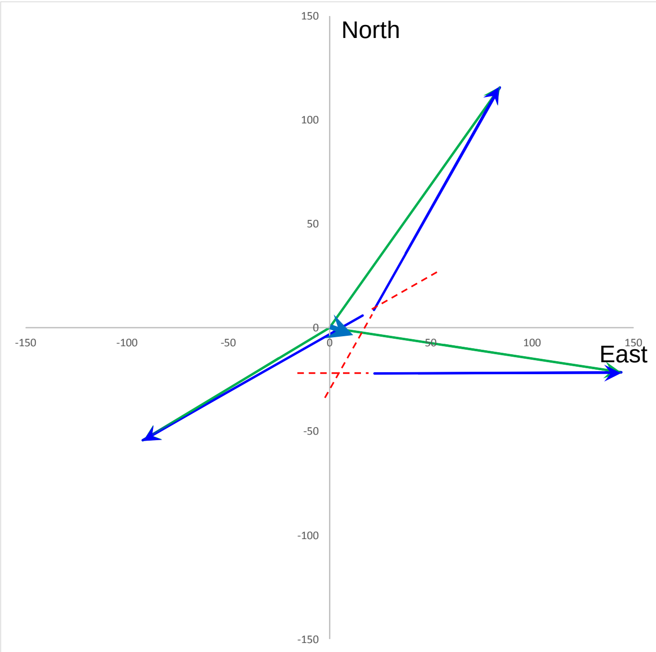


Figure 27. Better-posed solution

Figure 26 shows what is happening when only using a small amount of turn. The samples are not very independent, as they are all sort of pointing the same way. Very small changes of slope (heading) can make a very big difference in where the lines intersect. However, by spreading out the headings, as shown in figure 27, the solution is pushed back toward the middle where it should be. For best results, measurements should approximate complete turns.

So why haven't we done turn regression in the past? What changed to make it possible now? The ready availability of WAAS GPS gives high quality Time-Space-Position Information (TSPI) at minimal cost (compared to INS or ground based tracking radar). The method really needs an automatic recording system, which is now available through many EFIS without requiring an expensive DAS. The complex data reduction required is now made almost trivial by readily available software.

BUT WHAT ABOUT THE BEARHAWK?

This program did start out to find the Pitot-static corrections for the Bearhawk, but along the way morphed into an investigation of how to get a handle on uncertainty in Pitot-static testing. Figures 28 and 29 show that the Pitot-static corrections were actually determined.

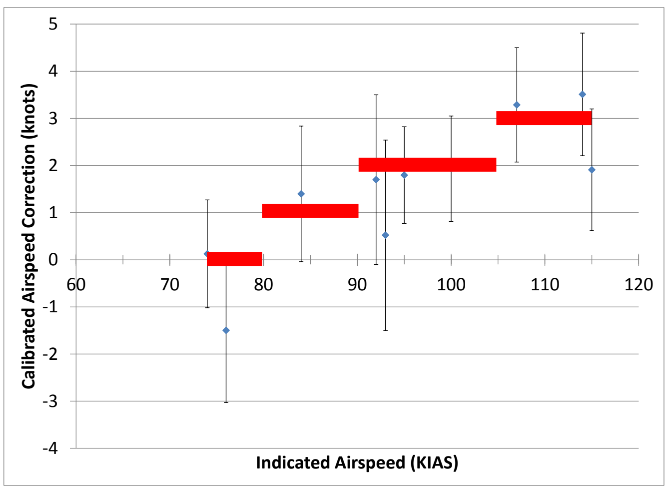


Figure 28. Calibrated Airspeed Correction

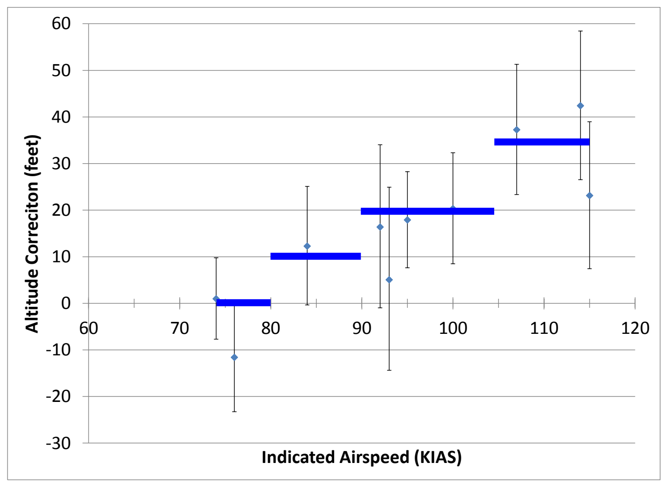


Figure 29. Altitude Correction

In figure 28, the data fairing was quantized to the nearest knot rather than drawn as a smooth curve. This was done because resolution below one knot was not needed. The confidence intervals provided by the turn regression method helped to make sense out of the data scatter with a small number of test points. The good news was that the airplane was slightly faster than I thought (positive correction), but the correction was not so large as to call the installation into question.

Figure 29 shows the altitude position correction calculated assuming a zero total pressure error. During this transition, the constant values of figure 28 gain a slight positive slope, which was again removed by quantizing the results to operationally significant values. In this case, variations less than 10 feet were ignored.

So did we achieve the desired properties listed earlier?

1) **Minimal special instrumentation**. No additional instrumentation was required beyond what was already installed in the aircraft
2) **Minimal external equipment**. No external equipment was required.
3) **Easy to fly**. A simple level turn or series of straight and level segments was sufficient. Where possible, the autopilot was used to fly the turns. Slight airspeed variations were okay, as actual airspeeds were used rather than one overall average airspeed. This assumed that the airspeed correction was essentially constant for slight airspeed variations.
4) **Quantifiable Uncertainty**. The output of the data reduction provides p values and confidence intervals to quantify the uncertainty.

Finally, because of the recording capabilities of the EFIS, handheld data were not required.

CONCLUSIONS

Uniform wind fields are unlikely to exist, especially near the ground.

The Turn Regression method can give good results with time-invariant winds that change with location.

Readily available automatic data recording systems collect sufficient data for tight confidence intervals with minimal piloting skills required.

Reasonably good results can be had from small sample sizes (handheld data) because the solution is well behaved.

For best results, measurements should approximate complete turns.

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AUTHOR

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Mr. Erb is a long time member of SFTE, having joined in 1983 as a charter member of the Texas A&M University student chapter, the very first student chapter ever. He has held positions in the Antelope Valley Chapter and is currently a Senior Member. He has previously presented three Symposium papers and three Symposium training classes.